Hexagonal Pixels and Indexing Scheme for Binary Images

For some purposes, this scheme is superior to rectangular pixels.

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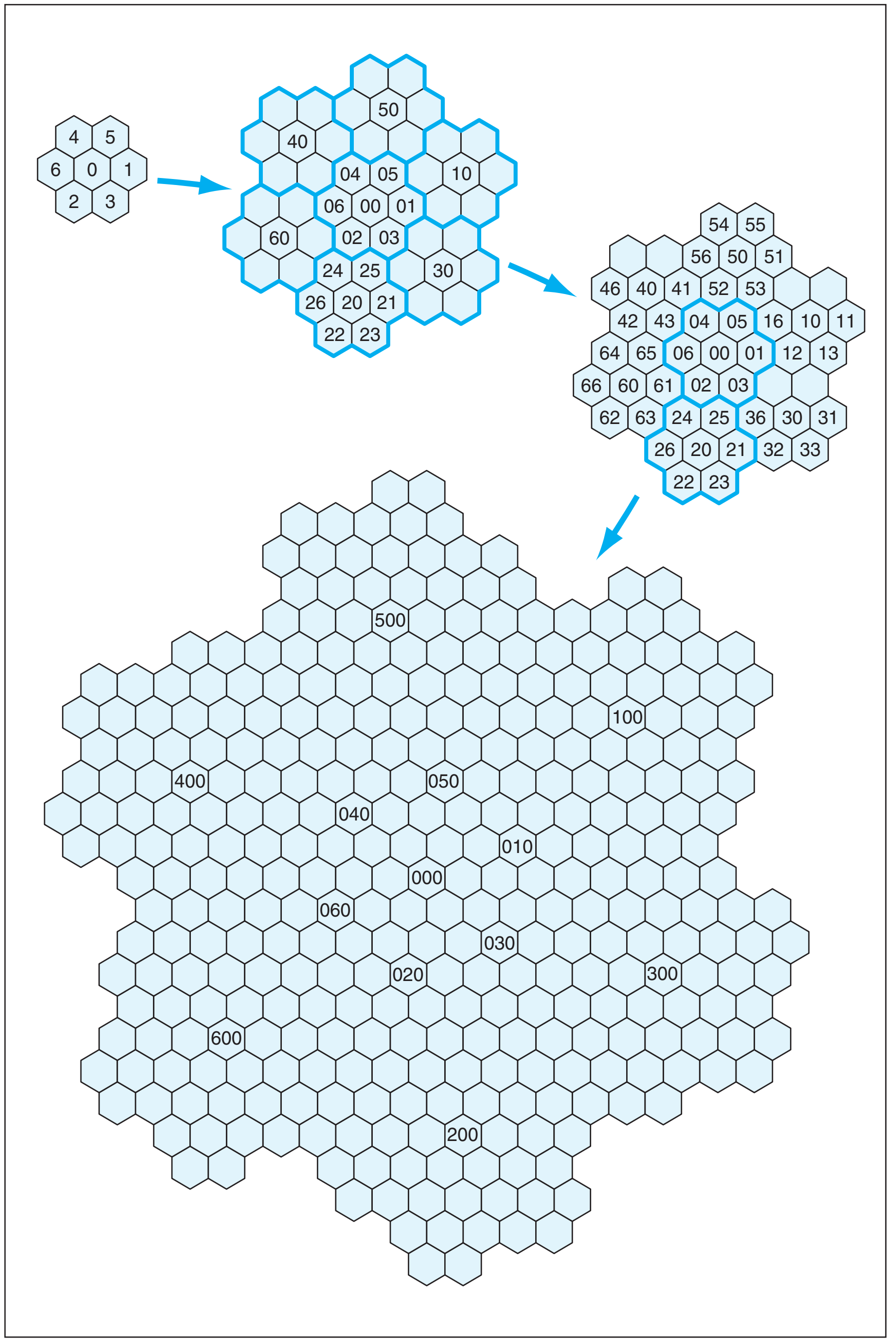
A scheme for resampling binary-image data from a rectangular grid to a regular hexagonal grid and an associated tree-structured pixel-indexing scheme keyed to the level of resolution have been devised. This scheme could be utilized in conjunction with appropriate image-data-processing algorithms to enable automated retrieval and/or recognition of images. For some purposes, this scheme is superior to a prior scheme that relies on rectangular pixels: one example of such a purpose is recognition of fingerprints, which can be approximated more closely by use of line segments along hexagonal axes than by line segments along rectangular axes. This scheme could also be combined with algorithms for query-image-based retrieval of images via the Internet.

A binary image on a rectangular grid is generated by raster scanning or by sampling on a stationary grid of rectangular pixels. In either case, each pixel (each cell in the rectangular grid) is denoted as either bright or dark, depending on whether the light level in the pixel is above or below a prescribed threshold. The binary data on such an image are stored in a matrix form that lends itself readily to searches of line segments aligned with either or both of the perpendicular coordinate axes.

The first step in resampling onto a regular hexagonal grid is to make the resolution of the hexagonal grid fine enough to capture all the binary-image detail from the rectangular grid. In practice, this amounts to choosing a hexagonal-cell width equal to or less than a third of the rectangular-cell width. Once the data have been resampled onto the hexagonal grid, the image can readily be checked for line segments aligned with the hexagonal coordinate axes, which typically lie at angles of 30°, 90°, and 150° with respect to say, the horizontal rectangular coordinate axis. Optionally, one can then rotate the rectangular image by 90°, then again sample onto the hexagonal grid and check for line segments at angles of 0°, 60°, and 120° to the original horizontal coordinate axis. The net result is that one has checked for line segments at angular intervals of 30°. For even finer angular resolution, one could, for example, then rotate the rectangular-grid image ±45° before sampling to perform



The **Tree-Structured Indexing Scheme** provides information on both locations and levels of resolution. In each group of seven pixels that constitute one pixel at the next coarser level of resolution, the zeroth pixel is the central one.

checking for line segments at angular intervals of 15°.

In the tree-structured pixel-indexing scheme, the smallest hexagonal pixels in nearest-neighbor groups of seven constitute larger pixels, which, in turn, are grouped into still larger pixels (see figure), and so forth, proceeding from the finest resolution to the coarsest. Each pixel is identified by a sequence of integers in order of increasing resolution. In other words, the first integer in an address denotes the coarsest pixel that contains the location in question, the second integer denotes whichever one of the seven subpixels of the coarsest pixel contains the location of interest, and so forth down to the last integer, which denotes the smallest hexagonal cell that contains the location of interest.

For some purposes — especially performing quick searches for images that match query images, it is useful to resample a binary image data from finer hexagonal grids onto coarser hexagonal grids. In such a case, each pixel at the next coarser level of resolution is made exactly hexagonal and considered dark if more than three of its seven component pixels are dark.

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MSC-22901

Finding Minimum-Power Broadcast Trees for Wireless Networks

Algorithms for identifying viable trees have been derived.

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Some algorithms have been devised for use in a method of constructing tree graphs that represent connections among the nodes of a wireless communication network. These algorithms provide for determining the viability of any given candidate connection tree and for generating an initial set of viable trees that can be used in any of a variety of search algorithms (e.g., a genetic algorithm) to find a tree that enables the network to broadcast from a source node to all other nodes while consuming the minimum amount of total power. The method yields solutions better than those of a prior algorithm known as the broadcast incremental power algorithm, albeit at a slightly greater computational cost.

It is not possible to give more than a highly abbreviated and oversimplified summary of the method within the space available for this article. However, to give meaning to even this brief summary, it is necessary to present some details of the underlying rules, simplifying assumptions, and mathematical constructs in the following two paragraphs.

Each node is equipped with an omnidirectional antenna. The minimum transmitter power that enables the jth node to send information to the ith node is proportional to r_{ij}^{α} where r_{ij} is the distance from node j to node i and α is a channel-loss exponent that usually lies between 2 and 4, the exact value depending on the nature of the signal-propagation medium. Any node in a network can be used to relay a signal to any other node. For the purposes of this method, only the transmitter power levels are calculated: the receivers are assumed to draw negligible power.

For a network of N nodes, one constructs a power or cost matrix, which is an N-by-N matrix $\mathbf{P}$, of which each element P_{ij} is proportional to the minimum power needed for communication between nodes i and j as described above. To represent connections between nodes of the network, one uses an N-element vector, denoted a cut vector, each element of which represents the location of an element in a row of the power matrix. Associated with each cut vector is a threshold vector, the elements of which are the power-matrix elements picked out by the cut vector. The cost of a cut is defined as the sum of the elements of the threshold vector. A cut is regarded as viable if it enables a broadcast to reach all nodes; otherwise, it is regarded as not viable. Another means of characterizing the network is the transfer matrix, which is an N-by-N matrix that is a function of the threshold vector.

One element of the present method is a construct denoted the viability lemma, which provides a necessary and sufficient condition for a cut to be viable. The viability lemma is implemented by equations that require power operations on the transfer matrix. In the case of a large network, power operations on a matrix can be computationally expensive. An alternative algorithm for determining viability does not require matrix power operations; instead, it requires an iterative procedure that involves examination of the rows of the transfer matrix and that is guaranteed to reach an indication of either viability or non-viability in $N-1$ or fewer iterations.

Another element of the present method is the connection matrix, which is a binary N-by-N matrix representation of a tree. The connection matrix is built iteratively from a cut that has been found to be viable by one of the means described above.

Yet another element of the present method is a stochastic tree-generation algorithm that can be used to generate a viable cut vector. This is an iterative algorithm that starts with a transmission from the source node to a randomly chosen destination node, followed by heuristic selection of the next destination node at each subsequent iteration, until all intended destination nodes are reached. This algorithm converges in $N-1$ or fewer iterations.

This work was done by Payman Arabshahi, Andrew Gray, Arindam Das, Mohammed El-Sharkawi, and Robert Marks II of Caltech for **NASA's Jet Propulsion Laboratory**. *Further information is contained in a TSP (see page 1).*

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